## Addition and Subtraction of Decimals

decimals as fractions Write decimals as fractions, find common denominators, add or subtract the fractions, and express the answers as decimals. This confirms that when adding or subtracting, one must compute with digits of the same place value.
pLACE-VALUE interpretation Students consider the place value of digits and what that means when adding or subtracting numbers.

## Multiplication of Decimals

decimals as fractions Write decimals as fractions, multiply, write the answer as a decimal, and relate the number of decimal places in the factors to the answer.
place-value interpretation Students see why counting decimal points make sense and use the short-cut algorithm: multiply the decimals as whole numbers and adjust the place of the decimal in the product.

Zeke buys cider for $\$ 1.97$ and donuts for $\$ 0.89$. The clerk said the bill was $\$ 10.87$. What did the clerk do wrong?
The cider is $\$ 1.97=\frac{197}{100}$ and the donuts are $\$ 0.89=\frac{89}{100}$. So the cost is $\frac{197}{100}+\frac{89}{100}=\frac{286}{100}=2.86$. In $1.97+0.89$, we add hundredths to hundredths ( $1.9 \underline{7}+0.8 \underline{9}$ ), tenths to tenths $(1 . \underline{9}+0.89)$, and ones to ones $(1.97+\underline{0} .89)$.

The clerk incorrectly added dollars and pennies (ones and tenths, tenths and hundredths).

We can look at a problem using equivalent fractions.

$$
0.3 \times 2.3=\frac{3}{10} \times 2 \frac{3}{10}=\frac{3}{10} \times \frac{23}{10}
$$

The product as a fraction is $\frac{69}{100}$, as a decimal 0.69 .
The 100 in the denominator shows that there should be two decimal places (hundredths) in the answer. The denominator of the fraction tells us the place value needed in the decimal.

Using the fact that $25 \times 31=775$ students reason about a related product: $2.5 \times 0.31$ ( 2.5 is a tenth of $25,0.31$ is a hundredth of 31 , so the product is a thousandth of 775 ) $=0.775$.

$$
\begin{aligned}
& 3.25 \div 0.5=\frac{325}{100} \div \frac{5}{10}=\frac{325}{100} \div \frac{50}{100}=325 \div 50=6.5 \\
& 37.5 \div 0.015=\frac{375}{10} \div \frac{15}{1,000}=\frac{37,500}{1,000} \div \frac{15}{1,000}=37,500 \div 15=2,500
\end{aligned}
$$

This makes a whole number problem with the same quotient as the original decimal problem.
The fraction approach explains why moving decimal places works.

$$
0 . 0 1 5 \longdiv { 3 7 . 5 } = 0 . 0 1 5 \times 1,0 0 0 \longdiv { 3 7 . 5 \times 1 , 0 0 0 } = 1 5 \longdiv { 3 7 5 0 0 }
$$

$\frac{1}{2}=0.5, \frac{1}{8}=0.125, \frac{12}{75}=0.16 . \frac{4}{25}=\frac{16}{100}=0.16$
In simplified fraction form $\frac{12}{75}=\frac{4}{25}$ has only factors of five $\left(\frac{4}{5 \times 5}\right)$ in the denominator.
$\frac{1}{3}=0.333 \ldots, \frac{2}{3}=0.666 \ldots, \frac{8}{15}=0.533 \ldots, \frac{3}{7}=0.42857142 \ldots$
In simplified fraction form $\frac{26}{150}=\frac{13}{75}=\frac{13}{3 \times 5 \times 5}=0.1733333 \ldots$.
$6 \%$ of $\$ 7.50=$ cost of tax
$1 \%$ of $\$ 7.50=\frac{1}{100}$ of $\$ 7.50=\$ 7.50 \div 100=0.075$
6 of the $1 \%$ 's will give me $6 \%$. So, $6 \%$ of $\$ 7.50=\$ 0.45$.
$20 \%$ of some number equals $\$ 2.50$
Find how many $20 \%$ s it takes to make $100 \%$. In this case we need five. So, $5 \times \$ 2.50$ gives us $\$ 12.50$.

Find what $\% 12$ is of 48 . Students can solve this by computing how many 12 s in 48 . It takes four, so the percent is $\frac{1}{4}$ of $100 \%$ or 25\%.

